Department of Mathematics and Statistics

Colloquium

A Mathematical Story: A Discrete Analogue of a Complex Problem

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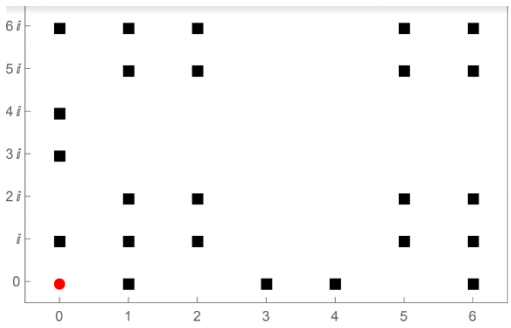
Mathematics includes the diverse areas of algebra, analysis, and geometry. Studying the interplay between these areas can lead to striking breakthroughs in our understanding of the deep structure of mathematics. Given a square matrix, we can calculate its determinant, trace, eigenvalues, and eigenvectors to deduce its core structural properties. A lesser known core invariant of the matrix is a set of complex numbers known as its numerical range. Just as a coin machine at Niagara Falls inputs two quarters and a penny and outputs a flattened penny with an impression of a waterfall scene, the numerical range function inputs a matrix and outputs a 2-dimensional image. If the matrix A has complex entries, then the numerical range of A is a convex set of complex numbers which contains the eigenvalues of A. While these complex numerical ranges have been completely classified for n-by-n matrices with n<5, little is known in higher dimensions. Roadblocks like these are common in stories of mathematical research. However, history tells us that the study of analogous problems can provide new insight on the original problem. Indeed, when we consider these matrices over finite fields, new simplifications and complications arise. Here we will discuss some pioneering work on these finite field numerical ranges with undergraduates at SUNY Geneseo, together with some generalizations using the machinery of number theory. It is foreseeable that a complete classification of finite field numerical ranges could be accomplished in the coming decade, which could in turn shed light on what a classification for complex matrices would look like.



Friday, November 9

4:00 pm

PS 306

For colloquium attendees, there will be light refreshments at 3:30 pm in PS 317