Mathematics Colloquium

Dr. Robert Fisher
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Friday, October 10, 2014
4:00pm
PS 324

Refreshments will be served in PS 317 at 3:30pm

Successive Apollonian Circles, a New Proof of the Similarity Fixed Point Theorem, and the Symmedians of a Triangle

Abstract

A similarity of the plane $\mathcal{E}$ is a mapping ϕ: $\mathcal{E} \to \mathcal{E}$ that either expands or contracts distances between points by a uniform factor $\delta > 0$. A similarity is called strict when its factor $\delta \neq 1$. The Similarity Fixed Point Theorem shows that a strict similarity has a unique fixed point. The talk will present a constructive proof of this theorem that quickly locates the fixed point using special circles, called *successive Apollonian circles*, that arise naturally from ϕ using the Apollonius Circle Theorem.

The second part of the talk will show that the symmedians of a triangle have a natural relation to the fixed points of strict similarities. Briefly given a scalene triangle ΔABC , there are two strict similarities, one preserving orientation and one reversing orientation, that send A to B and B to C . The talk will discuss why the fixed points of the two similarities are inverse to each other in the circumcircle C of ΔABC . The fixed point of the orientation preserving similarity lies inside C on the symmedian at vertex B , while the fixed point of the orientation reversing similarity is the point where the external symmedian at vertex B intersects the line l_{AC} .

In the talk the proof of the fixed point theorem and its relation to the symmedians of a triangle will be illustrated using the Geometer's Sketchpad. See the figure below.